CHAPTER 6 Study Guide and Review

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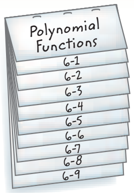
FOLDABLES Study Organizer GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Properties of Exponents (Lesson 6-1)

- The properties of powers for real numbers a and b and integers m and n are as follows.

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$
$a^m \cdot a^n = a^{m+n}$	$(a^m)^n = a^{mn}$
$(ab)^m = a^m b^m$	$a^{-n} = \frac{1}{a^n}, a \neq 0$

Operations with Polynomials (Lesson 6-2)

- To add or subtract: Combine like terms.
- To multiply: Use the Distributive Property.
- To divide: Use long division or synthetic division.

Polynomial Functions and Graphs (Lessons 6-4 and 6-5)

- Turning points of a function are called *relative maxima* and *relative minima.*

Solving Polynomial Equations (Lesson 6-6)

- You can factor polynomials using the GCF, grouping, or quadratic techniques.

The Remainder and Factor Theorems (Lesson 6-7)

- Factor Theorem: The binomial $x - a$ is a factor of the polynomial $f(x)$ if and only if $f(a) = 0$.

Roots, Zeros, and the Rational Zero Theorem (Lessons 6-8 and 6-9)

- Complex Conjugates Theorem: If $a + bi$ is a zero of a function, then $a - bi$ is also a zero.
- Integral Zero Theorem: If the coefficients of a polynomial function are integers such that $a_0 = 1$ and $a_n = 0$, any rational zeros of the function must be factors of a_n.

Key Vocabulary

degree of a polynomial (p. 320)
depressed polynomial (p. 357)
end behavior (p. 334)
leading coefficient (p. 331)
polynomial function (p. 332)
polynomial in one variable (p. 331)
quadratic form (p. 351)
relative maximum (p. 340)
relative minimum (p. 340)
scientific notation (p. 315)
simplify (p. 312)
standard notation (p. 315)
synthetic division (p. 327)
synthetic substitution (p. 356)

Vocabulary Check

Choose a term from the list above that best completes each statement or phrase.

1. A point on the graph of a polynomial function that has no other nearby points with lesser y-coordinates is a ____.
2. The _______ is the coefficient of the term in a polynomial function with the highest degree.
3. $(x^2)^2 - 17(x^2) + 16 = 0$ is written in ____.
4. A shortcut method known as ____ is used to divide polynomials by binomials.
5. A number is expressed in ________ when it is in the form $a \times 10^n$, where $1 \leq a < 10$ and n is an integer.
6. The __________ is the sum of the exponents of the variables of a monomial.
7. When a polynomial is divided by one of its binomial factors, the quotient is called a(n) _____________.
8. When we ________ an expression, we rewrite it without parentheses or negative exponents.
9. What a graph does as x approaches positive infinity or negative infinity is called the ________ of the graph.
10. The use of synthetic division to evaluate a function is called________.

Lesson-by-Lesson Review

6–1 Properties of Exponents (pp. 312–318)

Simplify. Assume that no variable equals 0.

11. $f^{-7} \cdot f^4$

12. $(3x^2)^3$

13. $(2y)(4xy^3)$

14. $\left(\frac{3}{5}c^2f\right)\left(\frac{4}{3}cd\right)^2$

15. MARATHON Assume that there are 10,000 runners in a marathon and each runner runs a distance of 26.2 miles. If you add together the total number of miles for all runners, how many times around the world would the marathon runners have gone? Consider the circumference of Earth to be 2.5×10^4 miles.

Example 1 **Simplify $(3x^4y^6)(-8x^3y)$.**

$(3x^4y^6)(-8x^3y)$

$= (3)(-8)x^{4+3}y^{6+1}$ Commutative Property and Product of Powers

$= -24x^7y^7$ Simplify.

Example 2 **Light travels at approximately 3.0×10^8 meters per second. How far does light travel in one week?**

Determine the number of seconds in one week.

$60 \cdot 60 \cdot 24 \cdot 7 = 604{,}800$ or 6.048×10^5 seconds

Multiply by the speed of light.

$(3.0 \times 10^8) \cdot (6.048 \times 10^5) = 1.8144 \times 10^{14}$ m

6–2 Operations with Polynomials (pp. 320–324)

Simplify.

16. $(4c - 5) - (c + 11) + (-6c + 17)$

17. $(11x^2 + 13x - 15) - (7x^2 - 9x + 19)$

18. $(d - 5)(d + 3)$

19. $(2a^2 + 6)^2$

20. CAR RENTAL The cost of renting a car is \$40 per day plus \$0.10 per mile. If a car is rented for d days and driven m miles a day, represent the cost C.

Example 3 **Find $(9k + 4)(7k - 6)$.**

$(9k + 4)(7k - 6)$

$= (9k)(7k) + (9k)(-6) + (4)(7k) + (4)(-6)$

$= 63k^2 - 54k + 28k - 24$

$= 63k^2 - 26k - 24$

6–3 Dividing Polynomials (pp. 325–330)

Simplify.

21. $(2x^4 - 6x^3 + x^2 - 3x - 3) \div (x - 3)$

22. $x^4 + 18x^3 + 10x^2 + 3x) \div (x^2 + 3x)$

23. SAILING The area of a triangular sail is $16x^4 - 60x^3 - 28x^2 + 56x - 32$ square meters. The base of the triangle is $x - 4$ meters. What is the height of the sail?

Example 4 **Use synthetic division to find $(4x^4 - x^3 - 19x^2 + 11x - 2) \div (x - 2)$.**

2	4	−1	−19	11	−2
		8	14	−10	2
	4	7	−5	1	0
	↓	↓	↓	↓	

The quotient is $4x^3 + 7x^2 - 5x + 1$.

Study Guide and Review

6–4 Polynomial Functions (pp. 331–338)

Find $p(-4)$ and $p(x + h)$ for each function.

24. $p(x) = x - 2$

25. $p(x) = -x + 4$

26. $p(x) = 6x + 3$

27. $p(x) = x^2 + 5$

28. $p(x) = x^2 - x$

29. $p(x) = 2x^3 - 1$

30. STORMS The average depth of a tsunami can be modeled by $d(s) = \left(\frac{s}{356}\right)^2$, where s is the speed in kilometers per hour and d is the average depth of the water in kilometers. Find the average depth of a tsunami when the speed is 250 kilometers per hour.

Example 5 **Find $p(a + 1)$ if $p(x) = 5x - x^2 + 3x^3$.**

$$\begin{aligned}p(a+1) &= 5(a+1) - (a+1)^2 + 3(a+1)^3\\ &= 5a + 5 - (a^2 + 2a + 1) + 3(a^3 + 3a^2 + 3a + 1)\\ &= 5a + 5 - a^2 - 2a - 1 + 3a^3 + 9a^2 + 9a + 3\\ &= 3a^3 + 8a^2 + 12a + 7\end{aligned}$$

6–5 Analyzing Graphs of Polynomial Functions (pp. 339–347)

For Exercises 31–36, complete each of the following.

a. Graph each function by making a table of values.

b. Determine the consecutive integer values of x between which the real zeros are located.

c. Estimate the x-coordinates at which the relative maxima and relative minima occur.

31. $h(x) = x^3 - 6x - 9$

32. $f(x) = x^4 + 7x + 1$

33. $p(x) = x^5 + x^4 - 2x^3 + 1$

34. $g(x) = x^3 - x^2 + 1$

35. $r(x) = 4x^3 + x^2 - 11x + 3$

36. $f(x) = x^3 + 4x^2 + x - 2$

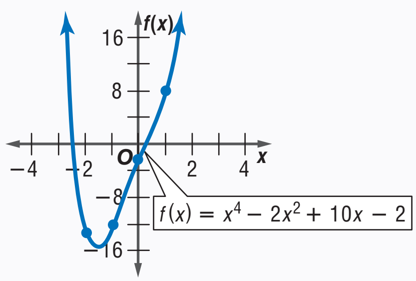
37. PROFIT A small business' monthly profits for the first half of 2006 can be modeled by (1, 550), (2, 725), (3, 680), (4, 830), (5, 920), (6, 810). How many turning points would the graph of a polynomial function through these points have? Describe them.

Example 6 **Graph $f(x) = x^4 - 2x^2 + 10x - 2$ by making a table of values.**

Make a table of values for several values of x.

x	−3	−2	−1	0	1	2
$f(x)$	31	−14	−13	−2	7	26

Plot the points and connect the points with a smooth curve.



Mixed Problem Solving
For mixed problem-solving practice, see page 931.

6–6 Solving Polynomial Equations (pp. 349–355)

Factor completely. If the polynomial is not factorable, write *prime*.

38. $10a^3 - 20a^2 - 2a + 4$

39. $5w^3 - 20w^2 + 3w - 12$

40. $x^4 - 7x^3 + 12x^2$ **41.** $x^2 - 7x + 5$

Solve each equation.

42. $3x^3 + 4x^2 - 15x = 0$

43. $m^4 + 3m^3 = 40m^2$

44. $x^4 - 8x^2 + 16 = 0$ **45.** $a^3 - 64 = 0$

46. HOME DECORATING The area of a dining room is 160 square feet. A rectangular rug placed in the center of the room is twice as long as it is wide. If the rug is bordered by 2 feet of hardwood floor on all sides, find the dimensions of the rug.

Example 7 **Factor $3m^2 + m - 4$.**

Find two numbers with a product of 3(−4) or −12 and a sum of 1. The two numbers must be 4 and −3 because 4(−3) = −12 and 4 + (−3) = 1.

$$\begin{aligned} 3m^2 + m - 4 &= 3m^2 + 4m - 3m - 4 \\ &= (3m^2 + 4m) - (3m + 4) \\ &= m(3m + 4) + (-1)(3m + 4) \\ &= (3m + 4)(m - 1) \end{aligned}$$

Example 8 **Solve $x^3 - 3x^2 - 54x = 0$.**

$x^3 - 3x^2 - 54x = 0$

$x(x - 9)(x + 6) = 0$

$x(x^2 - 3x - 54) = 0$

$x = 0$ or $x - 9 = 0$ or $x + 6 = 0$

$x = 0$ $\quad x = 9$ $\quad x = -6$

6–7 The Remainder and Factor Theorems (pp. 356–361)

Use synthetic substitution to find $f(3)$ and $f(-2)$ for each function.

47. $f(x) = x^2 - 5$ **48.** $f(x) = x^2 - 4x + 4$

49. $f(x) = x^3 - 3x^2 + 4x + 8$

Given a polynomial and one of its factors, find the remaining factors of the polynomial. Some factors may not be binomials.

50. $x^3 + 5x^2 + 8x + 4;\ x + 1$

51. $x^3 + 4x^2 + 7x + 6;\ x + 2$

52. PETS The volume of water in a rectangular fish tank can be modeled by the polynomial $3x^3 - x^2 - 34x - 40$. If the depth of the tank is given by the polynomial $3x + 5$, what polynomials express the length and width of the fish tank?

Example 9 **Show that $x + 2$ is a factor of $x^3 - 2x^2 - 5x + 6$. Then find any remaining factors of the polynomial.**

−2	1	−2	−5	6
		−2	8	−6
	1	−4	3	0

The remainder is 0, so $x + 2$ is a factor of $x^3 - 2x^2 - 5x + 6$. Since $x^3 - 2x^2 - 5x + 6 = (x + 2)(x^2 - 4x + 3)$, the remaining factors of $x^3 - 2x^2 - 5x + 6$ are $x - 3$ and $x - 1$.

Study Guide and Review

6–8 Roots and Zeroes (pp. 362–368)

State the possible number of positive real zeros, negative real zeros, and imaginary zeros of each function.

53. $f(x) = 2x^4 - x^3 + 5x^2 + 3x - 9$

54. $f(x) = -4x^4 - x^2 - x + 1$

55. $f(x) = 3x^4 - x^3 + 8x^2 + x - 7$

56. $f(x) = 2x^4 - 3x^3 - 2x^2 + 3$

DESIGN For Exercises 57 and 58, use the following information.

An artist has a piece he wants displayed in a gallery. The gallery told him the biggest piece they would display is 72 cubic feet. The artwork is currently 5 feet long, 8 feet wide, and 6 feet high. Joe decides to cut off the same amount from the length, width, and height.

57. Assume that a rectangular prism is a good model for the artwork. Write a polynomial equation to model this situation.

58. How much should he take from each dimension?

Example 10 **State the possible number of positive real zeros, negative real zeros, and imaginary zeros of $f(x) = 5x^4 + 6x^3 - 8x + 12$.**

Since $f(x)$ has two sign changes, there are 2 or 0 real positive zeros.

$f(-x) = 5x^4 - 6x^3 + 8x + 12$

Since $f(-x)$ has two sign changes, there are 0 or 2 negative real zeros.

There are 0, 2, or 4 imaginary zeros.

6–9 Rational Zero Theorem (pp. 369–373)

Find all of the rational zeros of each function.

59. $f(x) = 2x^3 - 13x^2 + 17x + 12$

60. $f(x) = x^3 - 3x^2 - 10x + 24$

61. $f(x) = x^4 - 4x^3 - 7x^2 + 34x - 24$

62. $f(x) = 2x^3 - 5x^2 - 28x + 15$

63. $f(x) = 2x^4 - 9x^3 + 2x^2 + 21x - 10$

64. SHIPPING The height of a shipping cylinder is 4 feet more than the radius. If the volume of the cylinder is 5π cubic feet, how tall is it? Use the formula $V = \pi \cdot r^2 \cdot h$.

Example 11 **Find all of the zeros of $f(x) = x^3 + 7x^2 - 36$.**

There are exactly three complex zeros. There are one positive real zero and two negative real zeros. The possible rational zeros are ±1, ±2, ±3, ±4, ±6, ±9, ±12, ±18, ±36.

$$\begin{array}{r|rrrr} 2 & 1 & 7 & 0 & -36 \\ & & 2 & 18 & 36 \\ \hline & 1 & 9 & 18 & 0 \end{array}$$

$$x^3 + 7x^2 - 36 = (x - 2)(x^2 + 9x + 18)$$
$$= (x - 2)(x + 3)(x + 6)$$

Therefore, the zeros are 2, −3, and −6.